Markovian and multi-curve friendly parametrisation of a HJM model used in valuation adjustment of interest rate derivatives.

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Motivation

XVA NEEDS A JOINT, SIMULABLE CURVE MODEL

- Valuation adjustments on interest-rate derivative portfolios require simulating the entire curve system, discounting and projection curves jointly, over long horizons and across collateralised and uncollateralised netting sets.
- The Heath–Jarrow–Morton framework prices consistently by construction, but a generic HJM volatility makes the forward-curve state infinite-dimensional and non-Markovian, which is computationally prohibitive in an XVA engine.
- The paper constructs a parametrisation that is simultaneously Markovian and consistent with the post-crisis multi-curve setting, and calibrates it on the relatively less liquid PLN market.

The XVA family and the CVA cube

WHAT THE ENGINE MUST PRODUCE

XVA adjusts a risk-free valuation for risks the primary valuation ignores: CVA (counterparty risk), DVA (own credit), FVA (funding, including liquidity buffers), ColVA (collateral effects), MVA (initial margin funding), KVA (regulatory capital), TVA (tax).

- The interest-rate model must generate the CVA cube, the three-dimensional array

$$\blacksquare = \{ (V_i,\ S_j,\ t_k) :\ i = 1, \ldots, I;\ j = 1, \ldots, J;\ k = 1, \ldots, K \},$$

  portfolio values by scenario by time node, for every netting set.
- Counterparty and own hazard rates are taken as given or bootstrapped from market data; collateralisation under CSA annexes now dominates derivative trading.

The general HJM framework

NO-ARBITRAGE DYNAMICS AND THEIR COST

Under the risk-neutral measure the instantaneous forward curve of an *m*-factor model follows

$$f(t,T) = f(0,T) + \sum_{i=1}^{m} \int_0^t \sigma_i(u,T) \int_u^T \sigma_i(u,s)\, ds\, du + \sum_{i=1}^{m} \int_0^t \sigma_i(u,T)\, dW_i^Q(u),$$

with the drift pinned down by the volatility structure alone.

- The current term structure is an input by construction, so no initial-curve calibration is needed.
- The cost: the state space of a general volatility function is infinite-dimensional, and instantaneous forwards are not observed in the market.

The Cheyette subclass

SEPARABLE VOLATILITY COLLAPSES THE STATE

Lemma (Cheyette). In an M-factor HJM model with volatility of the separable form

$$\sigma_k(t,T) = \sum_{i=1}^{N_k} \frac{\alpha_i^k(T)}{\alpha_i^k(t)}\,\beta_i^k(t), \qquad k = 1,\ldots,M,$$

the forward-rate dynamics are determined by $n = \sum_{k=1}^{M} N_k$ state variables which are Markov processes, and the short rate is the initial forward rate plus the sum of the state variables.

- Markovianity makes the specification implementable inside an XVA engine and opens the way to analytic or semi-analytic pricing of swaptions, caplets and floorlets, the calibration instruments.
- Accuracy and speed depend on the number of parameters in the elementary functions $\alpha_i(\cdot)$ and $\beta_i(\cdot)$.

Familiar models as special cases

VOLATILITY FUNCTION FORMS WITHIN THE CHEYETTE CLASS

Volatility form	Summands	Name
$\sigma(t, T) = c$	$N_1 = 1$	Ho–Lee
$\sigma(t, T) = c\, e^{-a(T-t)}$	$N_1 = 1$	Hull–White
$\sigma(t, T) = \beta(t)\, e^{-\int_t^T \lambda(u)\, du}$	general	General exponential Cheyette
$\sigma(t, T) = \sum_i \beta_i(t)\, (T - t)^{i-1}$	$N_1 \geq 2$	General Hull–White

The paper works inside this menu, combining exponential decay, polynomial-in-t scaling and piecewise-constant mean-reversion to trade off flexibility against parameter count.

The multi-curve reality

POST-CRISIS MARKET STRUCTURE

- After the financial crisis, IBOR-based deposit rates and OIS rates diverged abruptly, from a plateau of a few basis points to more than 200 bp at the peak, and FRA par rates detached from the corresponding forwards: discounting and projection require separate curves.
- The parametrisation accommodates the multi-curve extension via a multiplicative spread between the discount and projection curves, preserving fast discount-factor generation at each simulation node.
- Desired model characteristics also include replicating rich curve shapes and incorporating negative rates reasonably.

Candidate specifications

FIVE ONE-FACTOR CHEYETTE MODELS ON PLN DATA

	Volatility form	Parameters
Mod1	$(c + bt)\, e^{-a(T-t)}$	3
Mod2	$c\, e^{-a(T-t)}$ (classic Hull–White)	2
Mod3	$c_1\, e^{-a_1(T-t)} + c_2 e^{-a_2(T-t)}$	4
Mod4	$(c + bt)\, e^{-\int_t^T \lambda(u) du}$, piecewise-const. λ, node at 4	5
Mod5	Mod4 + constant summand d	6

Mod4 follows Beyna and Wystup; Mod3 stays within two summands of the classic Hull–White form.

Calibration design

SWAPTION VOLATILITIES, NELDER-MEAD, EXPLICIT QUALITY GATES

- Calibration targets the market swaption volatility surface (log-normal Black quotes) on selected dates of the 2014–17 period, with quality checked both in volatilities and in nominal prices per unit notional.
- The optimiser is brute force with a Nelder–Mead finish: derivative-based and Brent or Powell methods scale poorly in this dimension, and simulated annealing requires lengthy tuning.
- Each calibration passes through eight checks, four core (M1–M4) and four secondary (M5–M8), and is classified Good (all met), Passed (core met, at most one secondary breached) or Failed.

Calibration results

WHAT THE PLN MARKET CAN AND CANNOT IDENTIFY

- Data availability disciplines the model choice: with swaption volatilities only, and no active caps and floors market, a three-factor exponential specification cannot be calibrated on the PLN market.
- Hull–White with two summands (Mod3) achieves satisfactory calibration quality and computation time, with Good outcomes on the reported 2014–17 calibration dates.
- A caveat on the data: with rates near zero, log-normal Black volatilities are distorted close to negative strikes, which the quality checks in prices are designed to catch.

Guidance for the XVA engine

CONCLUSIONS FOR IMPLEMENTATION

- The calibration step can be scheduled ahead of scenario generation, and the XVA integral should be discretised no finer than the simulation's horizon steps.
- Overall performance depends on many degrees of freedom beyond the model choice: time-scale granularity, interpolation, and the minimisation method.
- For a less liquid market the recommended compromise is a low-parameter Markovian Cheyette specification with a multiplicative multi-curve spread, rather than a richer model the data cannot identify.

Thank you.

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