From point through density valuation to individual risk assessment in the discounted cash flows method.

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International Journal of Finance & Economics, 26 (2021), 5621-5635 · doi:10.1002/ijfe.2084

Motivation

A POINT ESTIMATE CONCEALS THE UNCERTAINTY THAT MATTERS

- The discounted-cash-flow method delivers a single number, $V = \sum_t \mathbb{E}[CF_t] \,/\, (1 + r)^t$, from inputs that are themselves uncertain: projected cash flows and the discount rate.
- Sensitivity tables and scenario grids, the standard practice, probe the input space pointwise and carry no probability content. PERT (US Navy, 1958) translated qualitative scenario labels into numbers, but only through the first two moments under a Gaussian assumption.
- The paper reformulates the valuation to deliver the full distribution of outcomes implied by explicit distributions on the inputs, and reads risk measures off that distribution.

Downside risk: the lineage

FROM SAFETY-FIRST TO VAR

- Roy (1952) posits that the investor prefers safety of principal first, requiring a minimum acceptable return, with the reward-to-variability ratio as the criterion, the concept that evolved into the Sharpe ratio.
- The downside-risk literature that followed produced below-mean semivariance, below-target semivariance, lower partial moments, Value-at-Risk and VaR-equivalent volatility.
- Judging a simulated valuation by its first two moments alone wastes the model-building effort: the object of interest is the density, and the downside measures are functionals of it.

Stochastic DCF: cash-flow decomposition

THEORETICAL SETTING

Cash flows are represented by revenue, cost, scaled and fixed components,

$$CF(t) = \sum_{i=1}^{N} R_i(t) - \sum_{i=1}^{M} C_i(t) + \sum_{i=1}^{K} S_i\big((R_n), (C_m), (Y_e), t\big) + F,$$

with explicit stochastic processes on the drivers and on the discount rate, so the valuation V becomes a random variable.

- Scaled variables S_i are functional forms of the directly simulated drivers: business-cycle-dependent prices, piecewise cost schedules, tax paths. They inherit randomness without their own SDEs.

The menu of driver processes

SDES MATCHED TO SIGN AND MEAN-REVERSION PROPERTIES

For strictly signed drivers (revenues, costs):

$$dX_i = \mu_i X_i \, dt + \sigma_i X_i \, dW_i \quad \text{(GBM)}, \qquad dX_i = \kappa_i(\theta_i - X_i)\, dt + \sigma_i \sqrt{X_i}\, dW_i \quad \text{(CIR)}.$$

For drivers of either sign:

$$dX_i = \mu_i \, dt + \sigma_i \, dW_i \quad \text{(ABM)}, \qquad dX_i = \kappa_i(\theta_i - X_i)\, dt + \sigma_i \, dW_i \quad \text{(OU)},$$

and for discontinuous events a pure jump process $dX_i = \kappa_i \, dJ_i(\lambda, t)$ with Poisson intensity λ and jump magnitude κ_i. Correlation across drivers is imposed on the Brownian increments.

From simulation to density

MONTE CARLO, KDE AND THE CORNISH-FISHER ALTERNATIVE

- The valuation density is generated by Monte Carlo simulation of the driver system and estimated by kernel density methods, at both the cash-flow and the valuation level.
- A Cornish–Fisher expansion (1938) is implemented as the analytic alternative, capturing skewness and kurtosis beyond the Gaussian approximation, with the known caveats on its validity region (Jaschke 2002).
- Discrete probability mass functions remain available where the analyst insists on hand-shaped scenarios, at the cost of losing the catalogue of named-distribution properties.

Individual risk assessment

FUNCTIONALS OF THE VALUATION DENSITY

The same density answers the pricing and the risk question:

- **Consistency check** against the risk-free benchmark valuation t_{rf},

$$E = \left| 0.5 - \tilde{F}_{SDCF}(t_{rf}) \right| ,$$

  which should be close to zero under the first asset-pricing theorem in a perfect-market limit.

- **Reservation valuations** at chosen confidence levels,

$$t_{\min,99\%} = Q(0.01, \tilde{V}_{SDCF}), \quad t_{\min,95\%} = Q(0.05, \tilde{V}_{SDCF}), \quad t_{\min,90\%} = Q(0.10, \tilde{V}_{SDCF}),$$

  each reader applying their own risk tolerance to one common object rather than a single aggregated point recommendation.

Case study: correlated drivers

A TWO-PRODUCT FIRM OVER A FINITE HORIZON

- Sales of product 1 follow GBM with $\mu_1 = 0.01$, $\sigma_1 = 0.5$, $X_1(0) = 120$; sales of product 2 follow CIR with $\theta_2 = 75$, $\kappa_2 = 0.1$, $\sigma_2 = 0.5$, $X_2(0) = 80$; a business-climate indicator follows OU with $\theta_3 = 0$, $\kappa_3 = 0.1$, $X_3(0) = 5$.
- The three processes carry the correlation matrix with $\rho_{12} = 0.75$, $\rho_{13} = 0.1$, $\rho_{23} = -0.25$.
- Independent of these, surprise regulatory-adjustment costs (a heavily regulated market) follow a pure Poisson process with intensity $\lambda = 0.07$, magnitude 10,000 and a 50/50 sign.

Case study: scaled variables

FUNCTIONAL FORMS ON TOP OF THE DRIVERS

- Prices of product 1 are business-cycle dependent: a base price of 40 plus the business-climate indicator scaled by $\alpha_{P_1} = 1$.
- Fixed costs follow a piecewise schedule in total sales TS: 13,000 if $TS \geq 17{,}000$, 9,000 if $TS < 12{,}000$, and 9,500 otherwise. Financial costs combine a base of 400 with a business-climate term scaled by $\alpha_{Int} = 50$ or -80.
- The tax rate declines deterministically, $tax(t) = tax_{base}(1 - \Delta_{tax})^t$, expressing a long-term view of falling taxes.

Computational performance

PARSIMONY AS A DESIGN GOAL

- The exercises simulate and rescale more than 15 variables in 300,000 paths within about 3 minutes on an average CPU: density valuation is feasible inside routine appraisal work, not only in dedicated risk systems.
- Parameters of government bond yield curves for the discount-rate process, frequently from the Nelson–Siegel–Svensson model, are freely available from central banks, which keeps recalibration cheap.
- The Cornish–Fisher route provides an even cheaper analytic approximation where the expansion is valid.

Implications

FOR VALUATION PRACTICE

- Reporting a valuation as a distribution disciplines the input assumptions: every scenario grid becomes a special case of an explicit probabilistic statement.
- Disagreements between valuations can be decomposed into disagreements about input distributions, which are auditable.
- Valuation and risk measurement collapse into one object, and the framework extends to any present-value object with uncertain flows and rates, from equity valuation to project finance.

Thank you.

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Dec, M. (2021). From Point through Density Valuation to Individual Risk Assessment in the Discounted Cash Flows Method. International Journal of Finance & Economics, 26, 5621-5635. doi:10.1002/ijfe.2084

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